Real Time Two-Way Coupling of Fluids to Deformable Bodies using Particle Method on GPU

Kazuhiko Yamamoto
Graduate School of Design / ADCDU and CDS, Kyushu University, Japan

1 Introduction

In this work, I propose a flexible two way coupling method between fluid and deformable bodies using Particle Method, that enable to conserve the momentum of the coupled system to increase the stability and enable to set large size time step. Two way coupling method consider the both effects that is fluid to solid, and solid to fluid simultaneously. To simulate these phenomena, I construct a novel method by expanding the MPS(Moving Particles Semi-Implicit) Method, and present the efficient implementation on GPU to execute at a real time rate.

2 Method Overview

This section presents the overview of my method to simulate two-way coupling dynamics between deformable body and fluid. My method is based on MPS(Moving Particles Semi-Implicit) Method which represents fluid or solid as the collections of many particles.
First, we compute the advection term and external forces based on the Navier-Stokes equations in fluid particles, and external forces in solid particles, and obtain the intermediate velocity and positions.

$$u_f^* = u_f^n + \nu\nabla^2 u_f^n + f_f \tag{1}$$
$$r_{f*}^{n+1} = r_f^n + \Delta t u_f^* \tag{2}$$

$$u_s^* = u_s^n + f_s \tag{3}$$
$$r_{s*}^{n+1} = r_s^n + \Delta t u_s^* \tag{4}$$

where u_f^*, u_s^* represents the intermediate velocity of fluid particle and solid particle respectively. r_{f*}, r_{s*} represents the intermediate position, f is the external forces of fluid particle and solid particle respectively, and Δt, ν denote the time step size, and viscosity coefficient respectively. The differential operators is discretized by replacing each differential operator using the corresponding model of the MPS method, which represent the interaction of the neighboring particles [$Koshizuka\ et\ al.$ 1999].
Second, we compute the Poisson equation for fluid pressure P_f^{n+1} to satisfy the incompressibility of fluids.

$$\nabla^2 P_f^{n+1} = -\frac{\rho_f}{\Delta t^2}\frac{n^* - n^0}{n^0} \tag{5}$$

where n^*, n^0 denote the current and initial density of particles respectively. ρ_f denotes the density of fluids. Here, in the case of that we compute the n^* of a fluid particle, we need to sum up the weighting function for all neighboring particles that include solid particles. We can solve this equation to obtain the pressure of fluid particles in an implicit manner using the iterative method such as Conjugate Gradient Method.
On the other hand, the governing equation for deformable bodies which are assumed as linear elastics is represented as

$$\rho_s \frac{du_s}{dt} = \lambda div(tr(e)I) + 2\mu div(e) \tag{6}$$

where λ, μ denote the lame's parameters, ρ_s is the density of solid, and e is the strain of particles. Using the first term of the right hand side of Eq(6), we compute the pressure of solid particles P_s^{n+1} for fluid particles.

$$P_s^{n+1} = \vec{P^s} \cdot \vec{N} \tag{7}$$

where $\vec{N}$ is the normal vector for the solid particle at the point of solid surface. $\vec{P^s}$ is computed by the first term of the right hand side of Eq(6).
Next, we compute the gradient of the pressures for all particles.

$$(\nabla P^{n+1})_i = \frac{3.0}{n^0}\sum_{j\neq i}[\frac{(P_j - P_i)}{|\vec{r_j} - \vec{r_i}|^2}(\vec{r_i} - \vec{r_j})w(i,j)] \tag{8}$$

where $\vec{r}$, $w()$ denotes the position of particle, and the weighting function of MPS method respectively. i, j denote the particle's index. P represents the pressure of particles. For these P, we use P_f^{N+1} for fluid particles, and P_s^{N+1} for solid particles. To solve Eq(8), if particle i is fluid particle, we consider all neighboring particles j that include solid particle. However, if i is solid particle, we discard the effect of solid particles, so consider fluid particles only.
Finally, we obtain the final velocity of particles u_f^{n+1}, u_s^{n+1} as follow.

$$u_f^{n+1} = u_f^* - \frac{\Delta t}{\rho_f}\nabla P^{n+1} \tag{9}$$
$$\rho_s u_s^{n+1} = \rho_s u_s^* + \Delta t(\lambda div(tr(e)I) + 2\mu div(e)) + \Delta t\nabla P^{n+1} \tag{10}$$

Using this equation, we update intermediate velocities and positions to final velocities and positions. Besides, we can conserve the momentum of coupled system simultaneously.

3 Implementation and Result

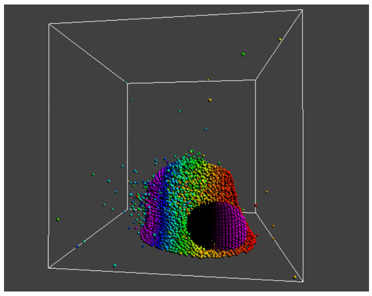

Fig.1 Interaction between fluid and elastic sphere

I implemented all of my method on GPU using NVIDIA CUDA. I used NVIDIA Geforce 9600M GT for GPU which is mobile GPU, and 40000 particles are used for the simulation. My implementation of this method uses 12 CUDA kernels. Each parameters of particles are stored in global memory continuously. To reduce the cost for searching the neighboring particles around a particle, first, each particles are stored in a grid space, and after that operation, search results are stored in continuous memory.
As the result of my implementation, the computational speed of about 40 FPS was archived. I set the time step size $0.01[s]$. This rate allows nearly real time simulation of two way coupling dynamics.

4 Conclusion

In this work, I presented a novel method to simulate momentum preserved two-way coupling system between fluids and deformable bodies using particle based simulation, and implemented on GPU to execute at a real time rate. In the future, I expand this method to enable to treat arbitrary boundary conditions.

References

KOSHIZUKA, S., and OKA, Y. 1996. *Moving-Particles Semi-implicit Method for Fragmentation of Incompressible fluid, Nucl. Sci. Eng, Vol.4, 29-46.*