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# Interactive Physically-Based Sound Design of 3D Model using Material Optimization (Appendix)

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## 1. Modal Sound Synthesis

Modal sound synthesis is based on the traditional linear modal analysis techniques [Adr91]. Modal analysis builds a reduced space for the elastic motion equation using of the solution of the generalized eigenvalue problem of the stiffness and mass matrices

$$KU = \Lambda MU \tag{1}$$

where $\Lambda$ is a diagonal matrix, containing the eigenvalues $(\lambda_1, ...\lambda_r)$, and $U$ is a matrix in which each column is a eigenvector $(u_1, ...u_r)$ corresponding to the eigenvalue of Eq. (1). We can decouple the system by retaining $r$ $(r << n)$ number of the eigenpairs that have larger energies into the form

$$\ddot{q} + C\dot{q} + \Lambda q = U^T f_{ext} \tag{2}$$

where $q$ is the generalized displacements, and $u = Uq$. The solution of Eq. (2) are a bank of modes that are attenuated sinusoids that has various frequencies and amplitudes. The $i$-th mode is represented as

$$\begin{aligned} q_i &= a_i e^{-d_i t} sin(2\pi f_i t + \theta_i) && (3) \\ f_i &= \frac{\sqrt{\lambda_i}}{2\pi} && (4) \end{aligned}$$

where $t$ is the time, $f_i$ is the frequency of the mode, $d_i$ is the damping coefficient, $a_i$ is the excited amplitude, and $\theta_i$ is the initial phase. In general, $\theta_i$ is ignored and then, $i$-th mode parameter is represented as $\phi_i = (f_i, d_i, a_i)$. As the result, a collection of mode parameters $(\phi_1, ..., \phi_r)$ determines the vibration specification of the structure, and modal sound synthesis uses it as the sounding wave.

## 2. Gradient Computation

For the gradient computation in our design problem, we need the derivative of each mode frequency and amplitude with respect to the reduced design parameter $z$. The mode frequencies and amplitudes are functions of the eigenpairs, and the derivatives of $k$-th eigenvalue $\lambda_k$ and eigenvector $u_k$ with respect to $z$ is represented as

$$\begin{aligned} \frac{\partial \lambda_k}{\partial z} &= u_k^T \left(\frac{\partial K}{\partial z}\right) u_k \\ \frac{\partial u_k}{\partial z} &= -(K - \lambda_k M)^+ \left(\frac{\partial K}{\partial z}\right) u_k \end{aligned} \tag{5}$$

where $superscript^+$ denotes the pseudo inverse (Moore-Penrose inverse). We approximate this pseudo inverse using the result of the already obtained generalized eigenproblem by $(K - \lambda_k M)^+ \fallingdotseq U(\Lambda - \lambda_k M)^+ U^T$. For the details of derivation of the derivative of eigenpair, please read [Lee07]. In addition, the derivative of the stiffness matrix $K$ with respect to $z$ is represented as

$$\frac{\partial K}{\partial z} = \sum_{e=1}^{M} \frac{\partial K}{\partial Y_e} \frac{\partial Y_e}{\partial z} = \sum_{e=1}^{M} \frac{\partial K}{\partial Y_e} \Phi_e \tag{6}$$

where all but 24 entries (3 dimensions $\times$ 8 nodes) in $\frac{\partial K}{\partial z}$ are zero, and $K$ is a linear function for $Y_e$, then the computational costs are cheap.

## 3. Hybrid Optimization Scheme of Evolutional Strategy and Gradient Descent Approach

We employ a hybrid optimization scheme [CLJ09] of evolutional strategy (we used CMA-ES [HMK03]) and gradient descent approach (we used Quasi-Newton method) for solve our design problem.This hybrid approach first updates the design parameter using the gradient informations for searching the local optima, and to escape from bad local optima, the evolutional strategy part generates the offspring by two characteristic variation operators, and additive Gaussian mutation alternatively.

$$\begin{aligned} q^g &= QuasiNewtonUpdate(z^g) && (7) \\ z^{(g+1)} &= q^{(g)} + \rho^{(g)} B^{(g)} D^{(g)} y^{(g)}, && (8) \\ B^{(g)} D^{(g)} y^{(g)} &\sim N(0, C^{(g)}) && (9) \end{aligned}$$

where $z^{(g)}$, $g$ and $\rho^{(g)}$ are the design parameters, iteration step and a global step size respectively. $y^{(g)} \sim N(0, I)$ are independent realizations of a normally distributed random vector with zero mean and covariance matrix equal to the identity matrix $I$, and $C^{(g)}$ denotes the covariance matrix which is computed using $q^{(g)}$ and $z^{(g)}$ (please read [CLJ09] for the details).

## References

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